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EFFECTS OF ROUTE CHARACTERISTICS ON COST OF RAIL TRANSPORTATION

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Introduction

Were it not for the intervening topography between distantly separated points land transportation could be conducted over straight, level paths. Since, however, topography must be reckoned with, all routes necessarily lie along devious and undulating paths varying in amount and intensity of sinuosity and undulation. In view of the many valuable contributions to the subject of route characteristics apparently little can be added at this late day, but as the science of transportation becomes more and more exacting, just so the need, especially for analytical and comparative purposes, of a more comprehensive and simple criterion of effect becomes more urgent.

Because distance is used to express the amount as well as the intensity of the horizontal departure from a straight path and the vertical deviation from a level surface, the length, the rise and fall, and curvature of path are all necessary to accurately define a land transportation line, with distance or length conveniently expressed in miles, rise and fall in feet, and curvature in degrees of arc. As resistance to motion is a direct factor of the mass moved, it can best be stated in pounds per ton of weight. Likewise the resistance to horizontal departure from a straight line being a function both of the mass or weight of body diverted and the amount of diversion, it may be expressed in pounds per ton of weight per degree of arc.

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The Premises

There are involved then, aside from the element of time, the six inseparably related quantities—distance, rise, fall, curvature, resistance to straight level motion, and resistance to curvature. From these we are to determine 1st, the total energy required for moving a given tonnage, and 2nd, the amount of wear on path and vehicle used. The following are the symbols to be used:

k = A constant, equivalent to the energy required to move at uniform speed one ton over a mile of straight level path.
c = A constant, equivalent to the energy required to divert horizontally one ton one degree from a straight line.
W = Tons of weight to be moved.
M = Miles of level gradients.
M′ = Miles of ascending gradients.
m′ = Miles of descending gradients which are of such rate or length that no braking is required to limit velocity to a safe maximum.
m = Miles of distance through which brakes are applied to limit or reduce speed.
M = Total distance in miles = *M* + M′ + m′ + m.
R = Total rise in feet in M′ (sum of ascents).
F = Total fall in feet in m and m′ (sum of descents).
f′ = Total fall in feet in the length m′.
f = Total fall in feet (gradient and velocity head) in the length m.
D = Degrees of curvature in level gradients *M*.
D′ = Degrees of curvature in ascending gradients M′.
d′ = Degrees of curvature in descending gradients m′.
d = Degrees of curvature on braked length of line m.
D = Total degrees of curvatures = *D* + D′ + d′ + d.

Energy Equations

The energy consumed or work required in moving a weight W over *M* miles of level gradients with *D* degrees of curvature occurring thereon is evidently the overcoming of the resistance W k and W c through the distance *M* and the degrees *D*, respectively—or

$$W\,k\,M + W\,c\,D \qquad (1).$$

The resistance of gravity due to grade alone on ascending gradients of length M′ and rise R is $W \frac{R}{M'} M'$ (longitudinal component of force $\frac{W R}{M'}$ multiplied by M′, the distance through which it acts), while the resistance of the D′ curvature is W c D′, so that the energy required for moving the weight W over all ascending gradients is

$$W k M' + W \frac{R}{M'} M' + W c D' \quad \text{or}$$

$$W k M' + W R + W c D' \qquad (2).$$

Up to the safe speed limit the fall of all descending gradients is an aid towards overcoming rolling and curve resistance, the amount of such aid being $W \frac{f'}{m'} m'$ (longitudinal component of force $\frac{W f'}{m'}$ multiplied by m′, the distance through which it acts), and as to whether it entirely overcomes rolling and curve resistance, the net result will be accurately obtained by the algebraic sum of the three, the former being negative and the two latter positive. The energy consumed on f′ gradients is clearly

$$W k m' + W c d' - W \frac{f'}{m'} m' \quad \text{or}$$

$$W k m' + W c d' - W f' \qquad (3).$$

Beyond the maximum safe speed limit the fall of accelerating descending gradients becomes a positive obstacle to restrained motion, for its effect must be counteracted continuously through the distance in which it is present. The accelerating effect to be counteracted is of exactly the same intensity as the resisting effect to motion in the opposite direction. This accelerating force then being in the nature of a hindrance instead of aid requires energy for its dissipation amounting to $W \frac{f}{m} m$ (longitudinal component of force

$\frac{W f}{m}$ multiplied by the distance m, through which it acts).

Just as the fall of descending gradients becomes an obstacle instead of an aid to motion after a certain definite velocity thereof is reached, so both rolling and curve resistance become an aid instead of an obstruction in dissipating the accelerating force. The overcoming of the first consumes energy, the amount of which is diminished by the other two, the net consumption thereof for the length of braked gradients m being

$$W \frac{f}{m} m - W k m - W c d \quad \text{or}$$

$$W f - W k m - W c d \qquad (4).$$

Adding together the foregoing quantities of energy required (1) on level gradients, (2) on ascending gradients, (3) on descending gradients on which no restriction of speed prevails, and (4) on braked portions of the line, results in a complete equation for total work required to move the weight W from one end to the other of a sinuous and undulating path — thus:

$$\begin{aligned} \text{Total Work} &= W k M + W c D + W k M' + W R + W c D' \\ &\quad + W k m' + W c d' - W f' + W f - W k m - W c d \\ &= Wk (M + M' + m' - m) + Wc (D + D' + d' - d) \\ &\quad + W (R - f' + f). \end{aligned}$$

In the preceding, $M + M' + m' - m = M - 2\,m$, $D + D' + d' - d = D - 2\,d$, and $- f' + f = - F + 2\,f$. By substitution of these values and further simplification in form, it becomes

$$\text{Total Work} = W\,[\,k\,(M - 2m) + c\,(D - 2d) + R - F + 2f] \quad (5).$$

This is a general equation applicable to any kind of power provided only k and c are known. It is axiomatic in form for with no wasting of energy in counteracting the acceleration of gravity, that is when rates of gradients with their resisting curves do not exceed the grade of repose of the vehicle, rise and fall if equal do not consume any more energy in their negotiation than a level line of same length. In such case, $m = 0$, $d = 0$, and $f = 0$, and Total Work =

W (k M + c D). Again, if there are no braking gradients and the fall exceeds the rise, the total work is less than that required for a level line of same length and curvature, for in such event the Total Work = W (k M + c D + R — F). With no curvature and no rise and fall, m, D, d, R, F, and f are each zero, and the result is an equation for a straight level path, Total Work = W k M.

Variation In Velocity

Thus far only the work required in moving the weight W over the M miles regardless of the time devoted to its accomplishment has been considered. Obviously the weight W cannot be moved from a standstill at one end of the line M and be brought to rest at the other end without an acceleration at the starting point and a retardation at the opposite terminus. These changes of velocity are similarly repeated for stops and alternate reductions and resumptions of running speed intermediate between the two termini, and since the matter of time has become a controlling factor in transportation, all accelerations must or at least should be effected by traction of the locomotive and desired retardation by brake application regardless of gradients.

It is quite clear that the quantity m with its attendant d and f can without vitiating the correctness of Equation (5) be ascertained and included therein for various maximum permissible speeds on different portions of the line according to actual operation thereover. Again, it is equally clear that the fall f of the braked gradient m must of necessity be augmented by the amount of any velocity head, $h = \frac{v^2}{2g}$, dissipated by retardation. Therefore in case of each reduction of speed either to zero or to any other amount effected by braking there must be added to f the fall of the gradient on which the reduction is made, an amount corresponding to the velocity head dissipated. In this connection it should be noted, Equation (4), that the slope of the braked gradient itself, and therefore f, is positive or negative depending on whether the gradient is descending or ascending, respectively, and that the dissipation of velocity head is always positive regardless of slope of gradient.

The fall or rise, $\pm$ f, of the braked gradient having been included in respectively the — F, the sum of all descents, or the + R, the sum of all the ascents, it must of course appear in double value or 2 f in Equation (5). By adding to the gradient value of f the velocity head dissipated in retardation, precise provision is made for the work necessary to accomplish the retardation and at the same time (because of a doubling of this amount of velocity head, 2 f, in the equation) also the work which had previously been necessary to accelerate the speed to the velocity prevailing at the instant the retardation began. In effect, the inclusion with gradient f of the velocity head of retardation to zero or any other amount automatically adds the work of acceleration accomplished subsequently to the last preceding brake application, or in other words, the work of acceleration previously accrued and not otherwise absorbed is added to other work when and as velocity is dissipated. With the inclusion in f of an amount equivalent to the velocity head dissipated in each speed reduction or stop, final as well as intermediate, Equation (5) gives the correct amount of total work involved including that required for accelerations and successive retardations.

Numerical Value of Constants

All the foregoing presupposes that k and c are constant for all velocities and all rates of curvature. We know that for rail transportation they are not. Freight train car resistance varies with speed, weight per car, climatological conditions, weight of rail, and character of track generally. Locomotive resistance, exclusive of tender, may amount to as much as 25 lbs. per ton of weight on drivers depending upon the type of engine. Passenger train car resistance varies quite similarly with freight, but with somewhat higher averages. Apparently it is impossible to mathematically coordinate rolling resistance of mixed general traffic with all the different kinds, condition and weights of cars, ranges of temperature, direction of prevailing winds, precipitation, varying conditions of track, and speed of trains. The results of an attempt to do so would perhaps be in greater error than to use an average resistance for a speed ranging from that attained on ascending gradients as the lower limit to the maximum permissible velocity as the upper limit.

Ordinarily the greater resistance of the locomotive will not perceptibly raise a weighted average for the trailing load, but with large power and short freight trains and with passenger trains the higher locomotive resistance will have a pronounced influence in raising the average train resistance as a whole. Assuming for example that all these considerations in the operation of a particular line result in adopting a resistance of six pounds per ton for freight trains and seven pounds per ton for passenger traffic trains, then k becomes 15.84 feet ($\frac{6}{2000} 5280 = 15.84$) for freight, and 18.48 feet ($\frac{7}{2000} 5280 = 18.48$) for passenger.

In general rail transportation, c too is a variable functioning with speed, spacing of axles, and radius of curvature, although in just what way and to what extent has never been accurately determined. However, since it varies within a slight range of value even for the widest practicable limits of radius and speed, no appreciable error prevails by assuming it to be a constant and assigning thereto an average value. It would accordingly seem sufficiently accurate, even for exacting purposes, to assign to c the ordinarily used average value of .04 feet for both freight and passenger trains.

Applying the above arbitrary numerical values of k and c for a particular case to W gross tons of freight trains and P gross tons of passenger trains moved, Equation (5) with different values for m, d, and f, depending on different speeds and stops for freight and passenger, becomes:

For Freight Traffic, Total Foot-Tons of Work $= W [15.84 (M-2m) + .04 (D-2d) + R-F+2f]$ (6).

For Passenger Traffic, Total Foot-Tons of Work $= P [18.48 (M-2m) + .04 (D-2d) + R-F+2f]$ (7).

If $W = 1$, (6) becomes

Foot-Tons Per Gross Ton Freight Trains $= 15.84 (M-2m) + .04 (D-2d) + R-F+2f$ (8).

If $P = 1$, (7) becomes

Foot-Tons Per Gross Ton Passenger Trains $= 18.48 (M-2m) + .04 (D-2d) + R-F+2f$ (9).

As ascending gradients are descending gradients in reverse direction, the energy required to move a gross ton over the line one way will rarely, if ever, be the same for the opposite direction. Then too any great preponderance of empty cars moving customarily in one direction may warrant a value of k different from that appropriate to the reverse movement. The length of braked grades m, with d the curvature occurring thereon, and the corresponding f must be determined for both freight and passenger in the two directions, resulting in two values each for Equations (6) and (7) or for Equations (8) and (9)—one for say northbound freight movement, a second for southbound freight, a third for northbound passenger, and the fourth for southbound passenger.

Manual Application of Mechanical Power

Values resulting from the foregoing equations are for the tonnage moved regardless of the number of trains required for the moving. The commonly accepted operating unit is the train-mile, while the unit of commodity produced is the ton-mile. The production of commodity ton-miles carries with it as an inseparably involved part the tare ton miles or in other words a certain amount of gross ton mileage is required to produce a net ton-mile of transportation. Hence it follows that the operating unit, the train mile, as a producing method is a variable depending upon the gross tons per train. Total train wages, varying directly with the number of trains required to handle a given total gross tonnage, are accordingly governed entirely by the gross tons in each train. The cost of train wages therefore cannot be coordinated with ton miles of lading or even gross ton miles except by fixing the train load capacity of the locomotive at the requisite speeds. But since within practical limits as heretofore indicated the energy required for moving traffic is governed not at all by maximum capacity of locomotives, it is erroneous to predicate train wages wholly on consumption of energy.

Either a reduction in the rate of ruling gradient without increasing the length of line or an increase in tractive power of locomotive by the addition of a helper or otherwise would of course effect a decrease in train wages and accordingly a reduction in the cost of producing a net ton mile of transportation, but such reduction of ruling gradient or increase in traction power could not affect the one-way cost of energy for its production. Trainmen and enginemen as well as locomotives must be returned to the terminal whence they were started, and unless there is a perfect balance between ruling gradients, train speeds, and total gross tonnage in both directions, an economic waste results. Due partially to these considerations and partially to the dual schedule of time and distance by which trainmen and enginemen are paid, it is evidently impossible to set up a fixed or even an approximately correct relation between physical characteristics of line and train wages.

Relation of Maintenance To Energy

Equation (5) with proper average values for the constants k and c gives the work required or consumption of energy attendant upon moving a weight W over a path of known physical characteristics. There remains to ascertain the effect on the path itself and the vehicles used in the moving of the weight W thereover—in other words, the effect of physical characteristics of line on the cost of maintenance of the path or track and the maintenance of the equipment.

The cost of maintenance of either track or vehicle varies directly as the intensity of their use which in turn is measured by the forces acting upon them. The wear on or damage to vehicle and track produced by rolling the one on the other is directly proportional to the force required to effect the rolling. For example, the more a wheel load depresses a track structure, the farther the wheel sinks into the rail and vice versa, and the higher the coefficient of journal friction, the greater will be the wear on track and vehicle and proportionately greater will be the required rolling force. It is therefore proper to conclude that the wear on vehicle and track due solely to the rolling of the one on the other is measured by the force required for motion regardless of its application. In case of locomotion, however, the force required for motion is applied by traction of the locomotive equally to the track and the body of the vehicle through draft rigging. Thence it is transmitted without abatement to the journals. Not only therefore is there wear on track and equipment produced by rolling exclusively, but also an equal additional amount, insofar as the cost of its replacement is involved, caused by the tractive force required for its production. The wear then on straight level track and on equipment is measured by the rolling resistance of cars and locomotive combined with an additional equal amount for the overcoming thereof. Similarly curve wear on track and vehicle is measured by the force necessary to overcome curve resistance, and since this force is applied to the track by the locomotive, the total force due to curvature and acting on track and equipment is the curve resistance augmented by an equal amount for its dissipation.

To the effects of rolling and curve resistances must be added the independent effects of gradients. The force applied to the trailing load by the locomotive through its traction on the track structure on an ascending gradient will be positive and equal to the longitudinal component of the force of gravity on the slope of the gradient. On the other hand, the longitudinal component of the force of gravity acting on the slope of a descending gradient is negative and may be sufficient to reduce to zero the traction forces, in which event the track and equipment would be subjected to only the rolling and curve effect. In addition to the gradient influence in braking for either restrained or retarded motion, the track and equipment are subjected to an algebraic combination of rolling and curve resistances and braking effect, which last must be assumed to be exactly the same as that of an equal amount of traction force.

The intensity of the various forces or effects acting on track and equipment in moving the load W over the curvature and different kinds of gradients of any line M miles long will, when multiplied by respective distances throughout which those effects are present, give the combined measure of resulting wear or maintenance in the performance. The wear or use of appliances in locomotion is therefore proportional to the mechanical work equivalent to the resisting effort to which such appliances are subjected.

Maintenance Equations

Using the symbols previously defined and for the sake of clarity segregating traction forces from those produced in rolling by expressing the former in the aspect of common fractions, the following detailed equations give the work equivalent to the resisting effort of track and equipment in moving the weight W from one end to the other of a line M miles long:

On Level Gradients

$$\frac{k\,M}{M}\,W M + \frac{c\,D}{M}\,W M + k\,W\,M + c\,D\,W \qquad (1a)$$

On Ascending Gradients

$$\frac{R}{M'}\,WM' + \frac{k\,M'}{M'}\,WM' + \frac{c\,D'}{M'}\,WM' + k\,W\,M' + c\,D'\,W \qquad (2a)$$

On Descending Gradients Other Than Braking Gradients

$$-\frac{f'}{m'}\,Wm' + \frac{k\,m'}{m'}\,Wm' + \frac{c\,d'}{m'}\,Wm' + k\,W\,m' + c\,d'\,W \qquad (3a)$$

As with the energy equations, so in these two equations the m's, d's, and f's will be different providing differences in maximum permissible speeds or in stops for freight and passenger prevail, and if W = 1 and P = 1,

$$\text{Freight Maintenance Per Gross Ton} = 31.68\ (M-m) + .08\ (D-d) + R-F + 2f \qquad (8a)$$

$$\text{Passenger Maintenance Per Gross Ton} = 36.96\ (M-m) + .08\ (D-d) + R-F + 2f \qquad (9a)$$

Indexes of Cost

Obviously, as was pointed out with respect to the equations for required energy, both freight and passenger maintenance per gross ton may vary with direction of movement, and as before there must be ascertained two values each for maintenance equations (6a) and (7a) or equations (8a) and (9a), one for northbound freight, a second for southbound freight, a third for northbound passenger, and a fourth for southbound passenger. Since, however, the same values of k, M, m, D, d, R, F, and f must necessarily be used in energy and maintenance equations corresponding in kind and direction only four values of m, d, and f need be ascertained from the profile in addition to the data commonly shown thereon, and once determined and applied as indicated in Equations (8), (9), (8a), and (9a), they immediately supply an index of cost of the energy and maintenance of moving a gross ton of either passenger or freight equipment in either direction over the physical characteristics of the route.

These indexes are of a permanent nature subject to change only with changes in maximum permissible speeds and in number and location of stops, and when weighted with gross tons of the two kinds of traffic in appropriate direction, give the correct proportions of total cost attributable thereto. For example, they make possible with little or no approximation the determination of the division of total costs of energy and maintenance of track and equipment as between directions of movement, between freight and passenger in either direction, and as between distance, rise and fall, and curvature.

Total Braking Foot-Tons = W (f—k m—c d), (10)

which subtracted from Equation (5) gives

Total Traction Foot-Tons = W [k (M—m)+c(D—d) + R—F+f] (11)

The totals of the two kinds, that is braking and traction, will when weighted by 1 and 6, the ratio hereinbefore suggested, and divided into the total cost of items expended for the production of energy during the period in which the gross tonnage was moved, give the unit costs per foot-ton of braking and traction respectively. These unit costs applied to the total foot-tons of braking and traction required to move an anticipated or actual gross tonnage of traffic over any proposed or revised line of known physical characteristics result in the cost of energy for the accomplishment.

The unit cost of a foot-ton of resisting effort of track or equipment is determined from the recorded costs of an operated line by dividing respectively the costs of maintenance of track and maintenance of equipment incurred in moving a definite volume of traffic by the total foot-tons as given by Equation (5a) when applied to the operated line. These resulting unit costs for maintenance afford the means of computing the total maintenance costs of any number of alternate lines of known physical characteristics or the economic value of reducing maintenance expenses by alignment and gradient revision through their application to the total foot-tons found by Equations (5a) for any line as proposed or revised and for any anticipated or actual volume of traffic.

The two sets of costs, one for production of energy and the other for maintenance, cover costs attendant upon the expenditures of the energy required to move traffic and the use of track and equipment for the performance. Unit costs, therefore, must be compiled separately from only those items of expense which vary directly with either the consumption of energy or replacing the wear occasioned by the use, and once correctly ascertained, they are of utmost utility in a variety of ways for analytical and comparative purposes, and will prevail until there is a radical change in labor and material costs.

Some items of cost, such as engine house expense, are

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Some items of cost, such as engine house expense, are incurred partially in the production of traction and braking power and partially in maintaining running gear independent of the exercise of tractive effort and braking by the locomotive, and proper proportions thereof as previously indicated should be included separately in the power and maintenance costs. In addition to the cost for train locomotives of fuel, water, and judicious proportions of lubricants, other supplies, and enginehousing, there should also be properly included as an incurred cost of energy the expense of maintaining power braking appliances on cars and a portion of the cost of locomotive repairs. Since, however, these two maintenance costs are impossible of segregation from other items of maintenance of equipment expense and are therefore of necessity included in the determination of a unit cost of maintenance, their omission from energy cost is requisite in order to avoid duplication.

Because of the uncertainty in allocating proper portions of combined items of maintenance of equipment expense separately to freight and passenger service, it will be more nearly correct to assume that replacement of the wear of equipment occasioned by resisting a foot-ton of passenger movement is equal in cost to that of a foot-ton of freight movement. In any event, the effect of the two on the track is the same, a fact which also urges the assumption.

In the list of maintenance items of expense there are some which are only partially involved in the use of the track and the equipment for train movement and others not at all. The proper criterion in the matter of maintenance costs is the inclusion of such items or portions thereof together with supervision therefor as vary directly with the use of track and equipment.

Scope of the Equations

Unfortunately, because of dual time-distance schedules and other inevitable working conditions, trainmen's and enginemen's wages cannot be uniformly coordinated with route peculiarities, but since train wages can on other premises be predicted with precision, there is no good reason for attempting an approximate coordination. The purpose herein has been to present a common measure of the inescapable and constantly prevailing effects of physical characteristics of route regardless of other elements affecting transportation costs.

Application of the equations to certain important phases of rail operation is illustrated in Appendixes A, B, and C.

Appendix A

A revision of alignment was contemplated in a certain operating district over which there was moved in a normal year the following gross train tonnage including locomotives:

	Southbound	Northbound
Freight	2,096,247	1,430,818
Passenger	715,778	715,778

For the movement of this gross tonnage the following costs were incurred:

Transportation Expense Variable with Physical Characteristics	=$179,885
Maintenance Expense Variable with Physical Characteristics	= 653,095
Total	$832,980

The immediately succeeding tabular statement gives the data taken from the profile with the argument of prevailing maximum permissible speeds and definite location of stops for freight and passenger movement:

Physical Characteristics

	M	D	R	F	m	d	f
S. B. Freight	135	8,663	2,014	3,272	37.14	904	1,781
N. B. Freight	135	8,663	3,272	2,014	19.66	1,946	930
S. B. Passenger	135	8,663	2,014	3,272	26.93	751	1,574
N. B. Passenger	135	8,663	3,272	2,014	18.98	1,954	1,071

By applying these profile data in the four typical equations (6), (7), (6a), and (7a), here repeated for convenience, the number of foot-tons in thousands for transportation and maintenance expense was found to be as shown in the next succeeding statement:

Freight Transportation Foot-Tons $= W\,[15.84\,(M-2m)+.04\,(D-2d)+R-F+2f]$ (6)

Passenger Foot-Tons Transportation $= P\,[18.48\,(M-2m)+.04\,(D-2d)+R-F+2f]$ (7)

Freight Maintenance Foot-Tons $= W\,[31.68\,(M-m) + .08\,(D-d) + R - F + 2f]$ (6a)

Passenger Maintenance Foot-Tons $= P\,[36.96\,(M-m) + .08\,(D-d) + R - F + 2f]$ (7a)

	1000 Foot-Tons		
	Braking	*Traction*	*Maintenance*
S. B. Freight	2,425,358	4,995,195	12,629,684
N. B. Freight	774,072	6,128,750	10,458,149
S. B. Passenger	748,704	1,882,138	4,664,883
N. B. Passenger	459,529	3,393,862	5,793,525
Totals	4,407,663	16,399,945	33,546,241

The transportation expense divided by total braking and traction foot-tons in thousands weighted in the ratio of 1 to 6, and the maintenance expense divided by total maintenance foot-tons in thousands, gives the cost incurred per 1,000 foot-tons thus:

$$\text{Cost per 1,000 Foot-Tons Braking} = \frac{179885}{4407663 + 98399670} = \$0.001750$$

$$\text{Cost per 1,000 Foot-Tons Traction} = 6 \times .001750 = \$0.010500$$

$$\text{Cost per 1,000 Foot-Tons Maintenance} = \frac{653095}{33546241} = \$0.019468$$

The physical characteristics of the revised line appear in the following statement:

Physical Characteristics—Revised Line

	M	D	R	F	m	d	f
S. B. Freight	121.8	2,830	1,605	2,863	40.04	398	1,856
N. B. Freight	121.8	2,830	2,863	1,605	21.55	476	980
S. B. Passenger	121.8	2,830	1,605	2,863	28.69	221	1,671
N. B. Passenger	121.8	2,830	2,863	1,605	19.60	377	961

Applying these data to Equations (6), (7), (6a), and (7a), there result separately the total foot-tons in thousands required for transportation and maintenance in moving the gross train tonnage over the revised line:

	1000 Foot-Tons		
	Braking	*Traction*	*Maintenance*
S. B. Freight	2,528,074	4,171,941	10,981,410
N. B. Freight	887,107	5,608,440	9,417,864
S. B. Passenger	813,261	1,598,911	4,104,319
N. B. Passenger	417,614	3,010,596	5,120,355
Totals	4,646,056	14,389,888	29,623,948

The reduced cost of operation and maintenance, exclusive of train wages, over the revised line will therefore be:

Braking	4,646,056 @	$0.001750	$ 8,131	
Traction	14,389,888 @	$0.010500	$151,094	
Total Energy				$159,225
Total Maintenance	29,623,948 @	$0.019468		576,719
Total Energy and Maintenance				$735,944

A comparison of these totals with those actually incurred in the operation of the original line gives the reduction in operating expenses which should be expected from the contemplated improvement in route characteristics:

Cost of Energy	$179,885 — $159,225 = $20,660
Cost of Maintenance	653,095 — 576,719 = 76,376
Total Saving for Tonnage Moved	$97,036

Appendix B

As an illustration in the determination of the economic value of the elimination of curvature, an operating district of severe physical characteristics was selected. Probably, because of the difference in location and number of stops for freight and passenger, the varying speeds thereof, and the use of helper locomotives with their return light to one terminal, no better selection illustrative of the cost effect of physical characteristics could be made.

A consideration of all the factors influencing train resistance in mixed general traffic leads to the adoption of an average of six pounds per ton for both freight and passenger in both directions. No ordinary elimination of curvature could reduce the length of the district or the rates of prevailing ruling gradients sufficiently to affect train wages. The economic value of the elimination of curvature in this case therefore depends wholly on the total cost per degree made up of the other items of operating expenses varying with the amount of curvature and incurred in moving the volume of traffic for which the cost of curvature is to be determined. This volume of traffic is as follows:

GROSS TONNAGE INCLUDING LOCOMOTIVES

Eastbound Freight	4,075,200
Westbound Freight	3,487,120
Eastbound Passenger	953,270
Westbound Passenger	953,270
Total	9,468,860

TOTAL OPERATING EXPENSES AFFECTED BY CURVATURE

Transportation		$ 322,631.65
Maintenance of Way and Structures	$339,819.95	
Maintenance of Equipment	486,395.46	
		826,215.41
Total		$1,148,847.06

PHYSICAL CHARACTERISTICS OF LINE

$k = 15.84$ $M = 86.85$ $D = 6376$ $c = .04$

	m	f	d	R	F
Eastbound Freight	52.19	2936	2407	2425	3200
Westbound Freight	19.35	2389	2905	3200	2425
Eastbound Passenger	50.60	3057	2378	2425	3200
Westbound Passenger	19.88	2401	3048	3200	2425

The foregoing quantities were assembled from the profile of the district and when applied to Equations (5) and (5a),

Transportation Foot-Tons $= W [k (M—2m) + c (D—2d) + R—F + 2f]$ (5)

Mantenance Foot-Tons $= W [2k (M—m) + 2c (D—d) + R—F + 2f]$ (5a)

result in the next succeeding statement:

	1000 FOOT-TONS		
	Braking	*Traction*	*Maintenance*
Eastbound Freight	8,203,378	11,690,933	26,539,944
Westbound Freight	6,855,778	15,246,739	27,789,138
Eastbound Passenger	2,059,063	2,875,549	6,489,138
Westbound Passenger	1,872,222	4,165,885	7,592,652
Totals	18,990,441	33,979,106	68,410,872

From these figures and the recorded total costs directly affected by curvature, and by other physical characteristics as well, unit costs for braking, traction, and maintenance foot-tons are computed thus:

$$\text{Cost per 1000 Foot-Tons Braking} = \frac{322631.65}{18990441 + 203874636} = \$0.001448$$

$$\text{Cost per 1000 Foot-Tons Traction} = 6 \times .001448 = \$0.008688$$

$$\text{Cost per 1000 Foot-Tons Maintenance} = \frac{826215.41}{68410872} = \$0.012077$$

Braking and traction foot-tons per degree of curvature $= \mp$ c W, that is, the amount will be either negative or positive, and maintenance foot-tons per degree of curvature will be either zero or 2 c W for curvature which respectively does or does not occur on a section of line braked in the direction in which W is moved—(Equations (1)—(4) and (1a)—(4a) q. v.).

In the following statement there are shown the values for c W and 2 c W in 1000 foot-tons per degree of curvature:

		1000 Foot-Tons Per Degree		
	Gross Tons	*Braking*	*Traction*	*Maintenance*
Eastbound Freight	4,075,200	−163	+163	+326
Westbound Freight	3,487,120	−140	+140	+280
Eastbound Passenger	953,270	− 38	+ 38	+ 76
Westbound Passenger	953,270	− 38	+ 38	+ 76

Applying to these quantities the unit costs of $ 0.001448, $ 0.008688, and $ 0.012077 per 1000 foot-tons of braking, traction, and maintenance respectively, the costs per degree of curvature are compiled thus:

Cost Per Degree of Curvature

	On Braked Line		Not on Braked Line	
	Eastbound	*Westbound*	*Eastbound*	*Westbound*
Transportation				
Eastbound Freight	−0.236	+1.416	+1,416	+1.416
Westbound Freight	+1.216	−0.203	+1.216	+1.216
Eastbound Passenger....	−0.055	+0.330	+0.330	+0.330
Westbound Passenger....	+0.330	−0.055	+0.330	+0.330
Maintenance				
Eastbound Freight	0.000	+3.937	+3.937	+3.937
Westbound Freight	+3.382	0.000	+3.382	+3.382
Eastbound Passenger....	0.000	+0.918	+0.918	+0.918
Westbound Passenger....	+0.918	0.000	+0.918	+0.918
Combined	$5.555	$6.343	$12.447	$12.447

Obviously the cost of operation of a degree of curvature on any section of line braked for both freight and passenger in eastbound movement was $5.56 for the gross tonnage moved, $6.35 on any section braked in westbound movement, and $12.45 on any section not braked in either direction. The cost per degree of curvature on a section of line braked in freight but not in passenger movement eastbound will be $5.94 and in westbound movement $6.73.

Appendix C

Information as to the following three things was desired for the operating district for which the cost of a degree of curvature was determined in Appendix B:

1st: The cost of maintenance of way and structures for passenger and freight separately.

2nd: The effect on operating expenses due to a lowering of the summit elevation 100 feet by means of a tunnel.

3rd: The effect on cost of power and maintenance of an increase of 100,000 revenue tons of freight westbound with empty cars therefor returned eastbound.

1. In Appendix B the total cost for maintenance of way and structures varying with use of track was given at $339,-819.95 for a total gross tonnage of 7,562,320 freight and 1,906,540 pasenger moved. It was also found in Appendix B that there were required for maintenance in the moving a resisting effort equivalent to the following:

	Maintenance 1000 Foot-Tons	
Eastbound Freight	26,539,944	
Westbound Freight	27,789,138	
		54,329,082
Eastbound Passenger	6,489,138	
Westbound Passenger	7,592,652	
		14,081,790
Total		68,410,872

Cost of maintenance of way and structures due to movement of passenger traffic will be $\frac{14081790}{68410872}$ 339819.95 = $ 69948.54.

The cost of maintenance of way and structures for the freight movement will be similarly $\frac{54329082}{68410872}$ 339819.95 = $269871.41.

2. A reduction of 100 feet in the summit elevation of

the operating district described in Appendix B carries with it the following reductions for both freight and pasenger:

Sum of ascents R	100.00
Sum of descents F	100.00
Length of line M	0.20
Total curvature D	95°
Length of braking grades eastbound m	1.00
Length of braking grades westbound m	0.50
Curvature on eastbound braking grades d	20°
Curvature on westbound braking grades d	75°
Fall of eastbound braking grades f	75
Fall of westbound braking grades f	75

Since the reduction of length of line is small, there will be no change in train wages. The effect on other operating expenses can be determined more readily than otherwise by computing the reduction in foot-tons of energy and maintenance for the tonnage moved, and applying thereto the respective unit costs found in Appendix B.

Substituting the above reductions in Equations (10), (11), and (5a) with $k = 15.84$ common to freight and passenger, the four values each for braking, traction, and maintenance in 1000 foot-tons appear as follows:

	1000 Foot-Tons Eliminated		
	Braking	*Traction*	*Maintenance*
Eastbound Freight	237,828	266,225	532,449
Westbound Freight	225,524	245,684	495,506
Eastbound Passenger	55,633	62,276	124,550
Westbound Passenger	61,105	67,708	135,456
Totals	580,090	641,893	1,287,961

Applying to these totals their respective unit costs per 1000 foot-tons of braking, traction, and maintenance as developed in Appendix B, the reduction in operating expenses accompanying a 100 foot lowering of the summit elevation would be

Braking			
580,090 foot-tons	@	$0.001448	= $ 839.97
Traction			
641,893 foot-tons	@	$0.008688	= $ 5,576.76
Maintenance			
1,287,961 foot-tons	@	$0.012077	= $15,554.70
Total			$21,971.43

3. The 100,000 tons of revenue freight are to be loaded in cars of such weight and capacity that the average tare is 65 percent of the lading or in other words the gross tons of tare and lading westbound are 165,000 and of tare eastbound are 65,000. The car accountant's records show that for every ton of trailing load there are .2026 tons of locomotive, including helpers, eastbound and .2310 tons of locomotives, including helpers returned light, westbound—hence

Additional gross tonnage eastbound =	78,169
Additional gross tonnage westbound =	203,115
Total	281,284

Since foot-tons are directly proportionate to gross tonnage, there will be required for the additional gross tonnage to be moved the following foot-tons of braking, traction, and maintenance. (See Appendix B).

1000 Foot-Tons

Braking

Eastbound Freight $= \frac{78169}{4075200}\ 8203378 =$ 157,350

Westbound Freight $= \frac{203115}{3487120}\ 6855778 =$ 399,330 — 556,680

Traction

Eastbound Freight $= \frac{78169}{4075200}\ 11690933 =$ 224,250

Westbound Freight $= \frac{203115}{3487120}\ 15246739 =$ 888,080 — 1,112,330

Maintenance

Eastbound Freight $= \frac{78169}{4075200}\ 26539944 =$ 509,080

Westbound Freight $= \frac{203115}{3487120}\ 27789138 =$ 1,618,640 — 2,127,720

Unit costs per 1000 foot-tons are—braking \$ 0.001448, traction \$ 0.008688, and maintenance \$ 0.012077, (Appendix B), and the cost of energy and maintenance for the 100,000 revenue tons westbound with empty cars therefor returned eastbound will be:

556,680	@ \$0.001448	=	\$ 806.07
1,112,330	@ \$0 008688	=	\$ 9,663.92
2,127,720	@ \$0.012077	=	\$25,696.47
Total			\$36,166.46